# Two Interconnected Balloons: How Does the Air Flow?

Wenze Shi

*Nanjing Foreign Language School*

# Summary

Most people may assume that when two partially inflated balloons are connected together, the air will always flow from the larger balloon to the smaller one. This argument seems reasonable and corresponds to our instinct. But is that really so? It is proposed by researches that under some initial conditions, the air may flow from the smaller balloon to the larger one. This article will focus on this issue by theoretical and experimental approaches.

In this article, we will use **thermodynamic methods** and **material science** to analyze the cause of the phenomenon and determine the conditions in which one direction of air flow occurs by quantitative analysis. We will use experiment to further test our result and to implement theoretical analysis.

The special characteristics of rubber plays a big part in the phenomenon, and this article reveals those materialistic attributes of rubber that may be beneficial to material science and technological development.



**Key words:** air pressure, Jame-Guth theory, ideal rubber, T-connector

# Contents

# 1. Restatement and analysis of the problem

Two rubber balloons are partially inflated with air and **connected together by a hose with a valve**. It is found that **depending on initial balloon volumes**, the air can **flow in different directions**.

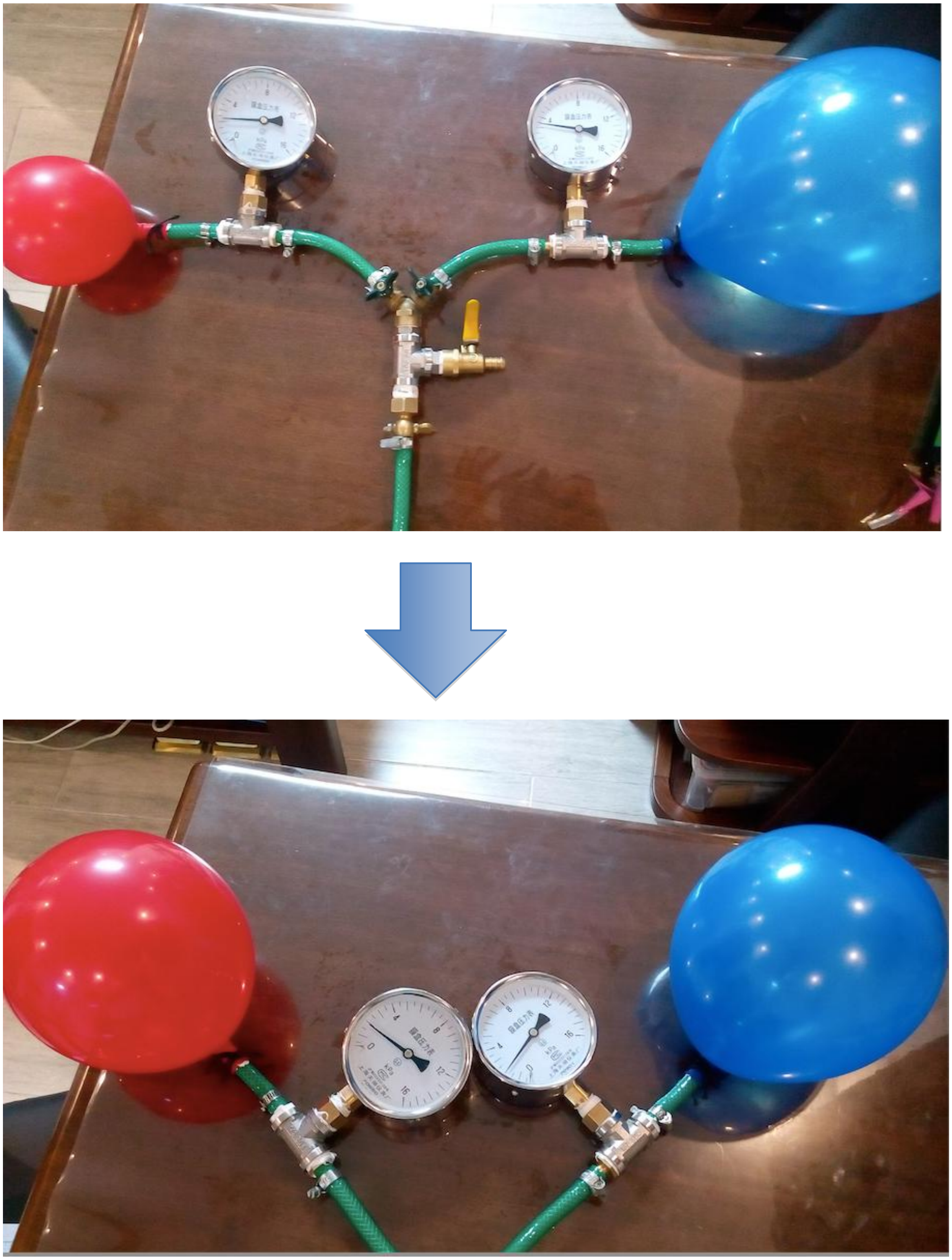
It is easy to see that the reason that air flows between the two balloons is that there is a difference in air pressure in the two balloons. We need to find the relationship between the radius of the balloon and the air pressure inside it so that we can use it to determine air flow at given volumes.

We will focus our theoretical research on the value of pressure as dependent on the materialistic property of rubber and thermodynamic laws. We will the use experiment to test the validity of pressure curve and analyze three possible scenarios: big to small, small to big, and equilibrium.

# 2. Preliminary experiment

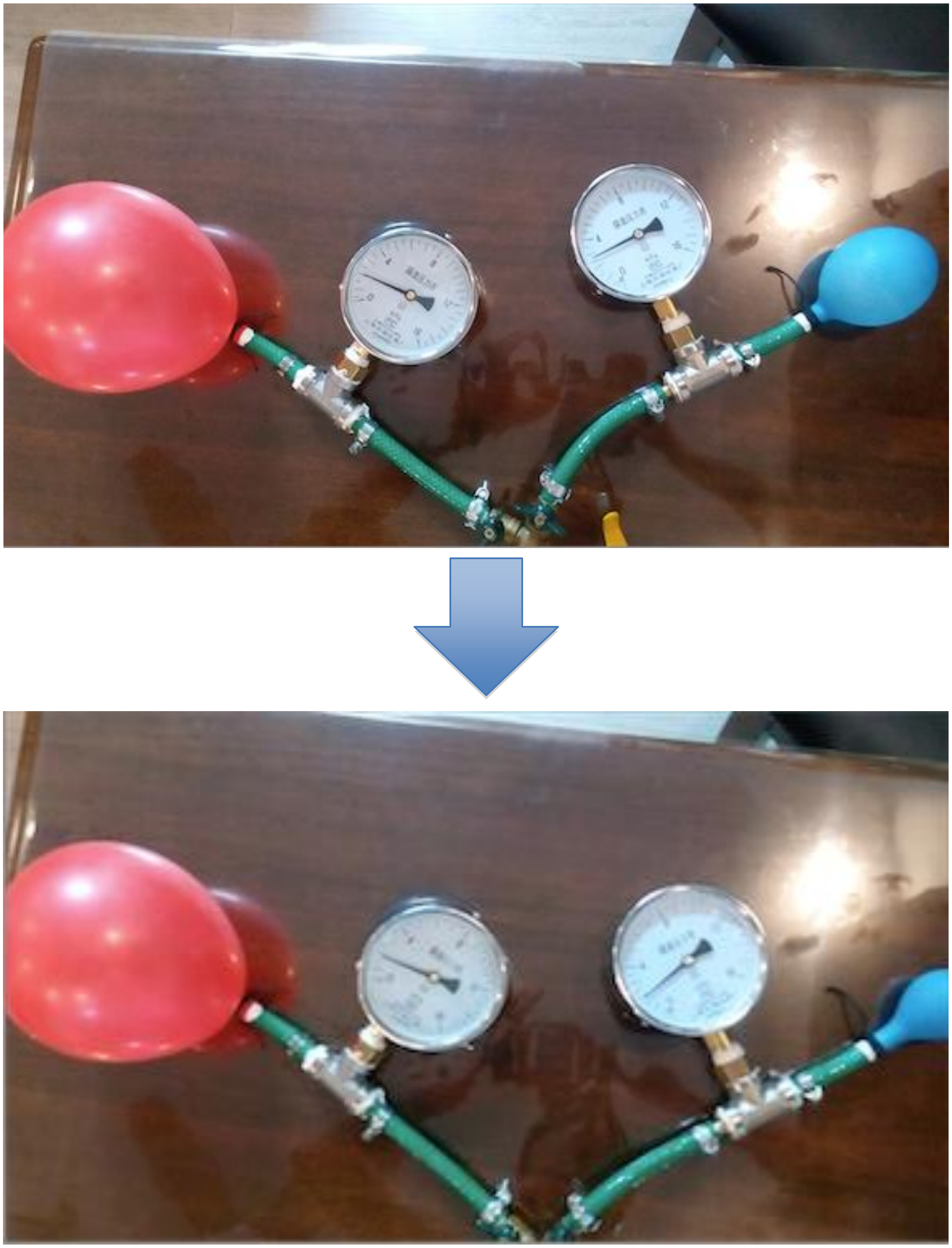
We observe the following phenomena, and found out that under initial volumes the big balloon shrinks, but on other conditions the reverse happens.

1. big to small:



Pic 1. The big balloon shrinks and the small balloon expands until their volumes are equal

2. small to big



Pic 2. The small balloon shrinks almost to initial volume while big balloon expands. The difference in volumes increases.

# 3. Model assumption and symbols

In this article we make the following assumptions:

1. We assume that the balloons are **ideal rubber balloons** in order to simplify calculation. In the case of non-ideal balloons we will discuss briefly in the following parts.
2. Since the balloons are not perfect spheres, we assume that the “radius” of the balloon is its **equivalent radius**, which satisfies $V = \frac{4}{3}\pi r^3$.
3. We assume that the process is quasi-static, and that the temperature remains that same throughout the process.

Here are the definitions of important symbols used in the article.

| V | Volume of the balloons |
|---|---|
| R | Radius of the balloons |
| d | Thickness of balloon shells |
| k | Boltzmann’s constant, $k = 1.38\times10^{-23} J/K$ |
| p | Pressure |
| F | Force |
| C | Other constants |
| T | Absolute temperature |

# 4. Theoretical analysis

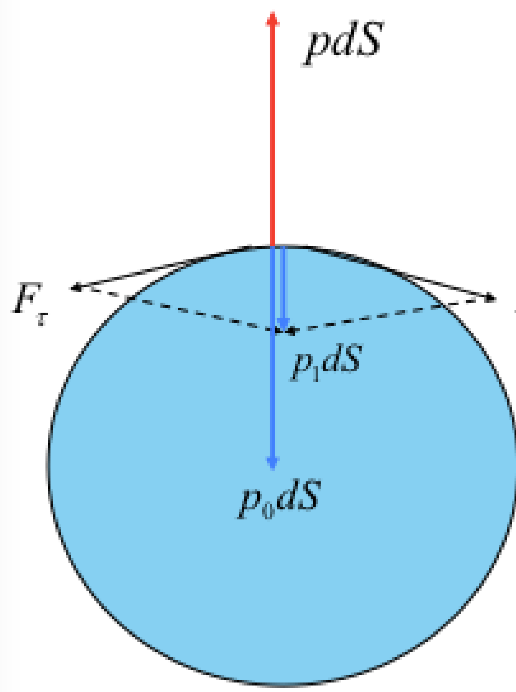
## 4.1 Determine the pressure curve

It is obvious that the air flows from the balloon with the bigger pressure to the one with the smaller pressure.

We choose an infinitesimal "patch" with area $dS$ from an ideal rubber balloon with radius $R$ and analyze the forces acting on the patch.

First of all, there exists atmospheric pressure $p_0 dS$ that points to the center of the sphere.

Also, surface tension $F_\tau$ is provided by the patches surrounding the chosen patch, and because of symmetry, the combined force of surface tension points to the center of the sphere as $p_1 dS$. To reach equilibrium, the air pressure inside the balloon must satisfy $pdS = p_0 dS + p_1 dS$, and therefore we have $p = p_0 + p_1$.




Pic. 3

Since $p_0$ is the same for both balloons, the value of $p_1$ determines the direction of air flow. We can find out the relationship between $p_1$ and $R$ by the following method.

Here we assume that the patches on the balloon are **ideal parallelepipeds**.

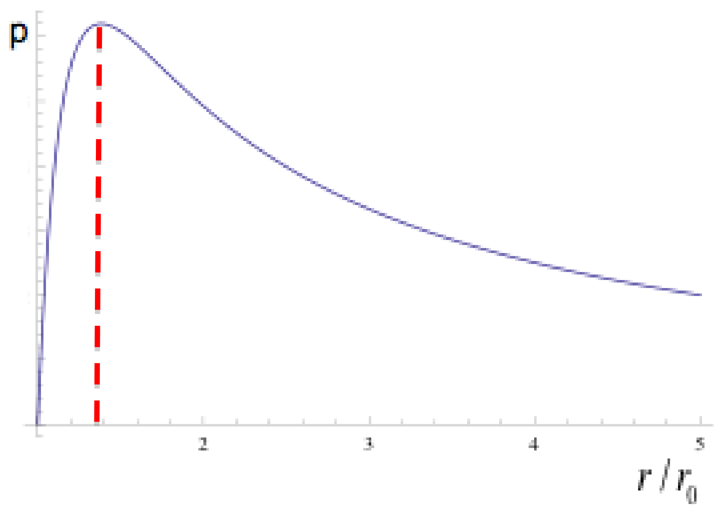
By James-Guth equation we have $f_i = \frac{1}{d_i}[kKT(\frac{d_i}{d_0})^2 - pV]$. Here, $f_i$ denotes the perpendicular (pointing towards the center) forces. $d_i$ and $d_0$ denote the final and initial thickness of the shell of the balloon. $k$ denotes Boltzmann's constant, and $K$ is a constant dependent on the number of possible structures inside an object.

Since it is hard to change the volume of rubber, we can assume that $V$ is constant, so the equation can be rewritten as $f_i = \frac{C_1}{d_i}(\lambda_i^2 - C_2 p)$, with $C_1 = kKT$, $C_2 = \frac{p}{kKT}$ and $\lambda_i = \frac{d_i}{d_0}$. On an ideal rubber balloon, the forces acting on a "patch" should be tangent to the surface, so the perpendicular force should equal to zero, so we have $\lambda_i^2 = \left(\frac{d_i}{d_0}\right)^2 = C_2 p$.

Because the volume of rubber can be approximately seen as $V = 4\pi R^2 d$, it is obvious that $d \propto \frac{1}{R^2}$. Therefore, $\frac{d_i}{d_0} = \left(\frac{r_0}{r_i}\right)^2$, $p = \frac{1}{C_2}\left(\frac{r_0}{r_i}\right)^4$

As a result, the tangential force $f_t \propto \left(\frac{r_i}{r_0^2}\right)\left[1 - \left(\frac{r_0}{r_i}\right)^6\right]$. Therefore the pressure $p = \frac{f_t}{\pi R^2} = \frac{C}{r_0^2 r_i}\left[1 - \left(\frac{r_0}{r_i}\right)^6\right]$. Differentiate the equation and we can find that the maximum pressure occurs when $r_i = \sqrt[6]{7} r_0 \approx 1.38 r_0 = r_p$.

Based on the function above, we obtain the following diagram of the p-r relitionship for an **ideal rubber balloon.**




Pic. 4

## 4.2 Analysis

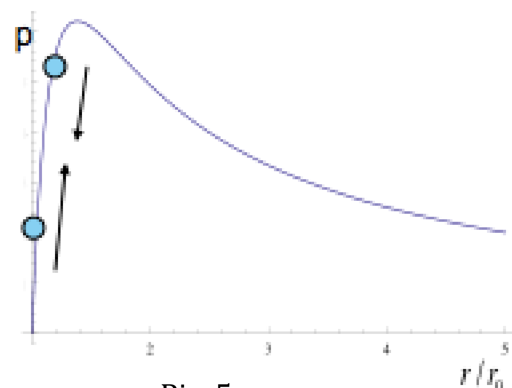
Based on the diagram above, we can see that as the balloon expands, the air pressure rapidly increases until reaching a maximum, then steadily decreases. The diagram corresponds with our daily experience.

When released, three possible situations will occur:

(i) Air flows from the bigger balloon to the smaller one. ($p_{big} > p_{small}$)

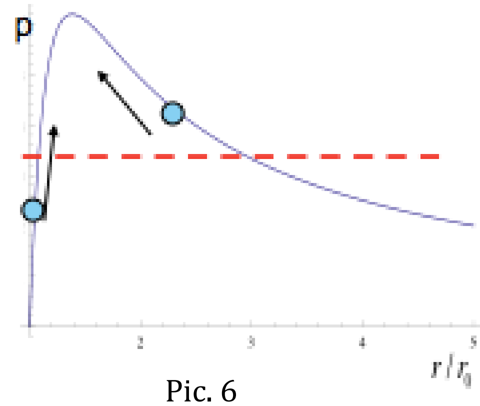
This will happen under the one of the following conditions:

(1) $r_{small} < r_{big} < r_p$



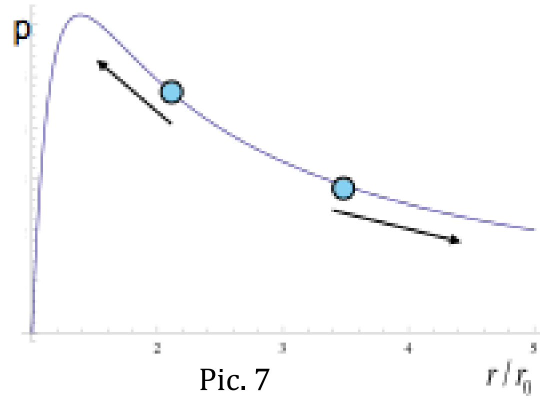

Pic. 5

(2) $r_{small} < r_p < r_{big}, p_{big} > p_{small}$




Pic. 6

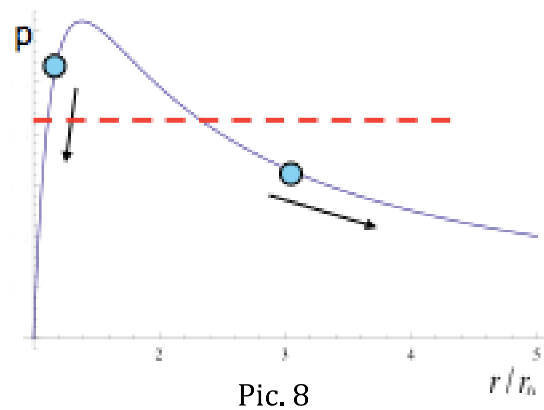
(3) $r_p < r_{small} < r_{big}$




Pic. 7

(ii) Air flows from the smaller balloon to the bigger one. ( $p_{big} < p_{small}$)

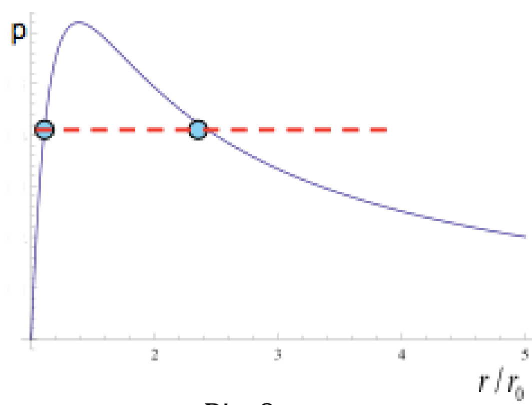
This will happen when $r_{small} < r_p < r_{big}, p_{small} > p_{big}$




Pic. 8

(iii) No air flow. ( $p_{big} = p_{small}$)

We will then try to calculate the relationship between the radii and the volumes of the two balloons:




Pic. 9

From $P=\dfrac{f_t}{\pi R^2}=\dfrac{C}{{r_0}^2 r}\left[1-\left(\dfrac{r_0}{r}\right)^6\right]$, we can obtain $\dfrac{{r_A}^7-{r_B}^7}{r_A-r_B}=\dfrac{{r_A}^6{r_B}^6}{{r_0}^6}$,

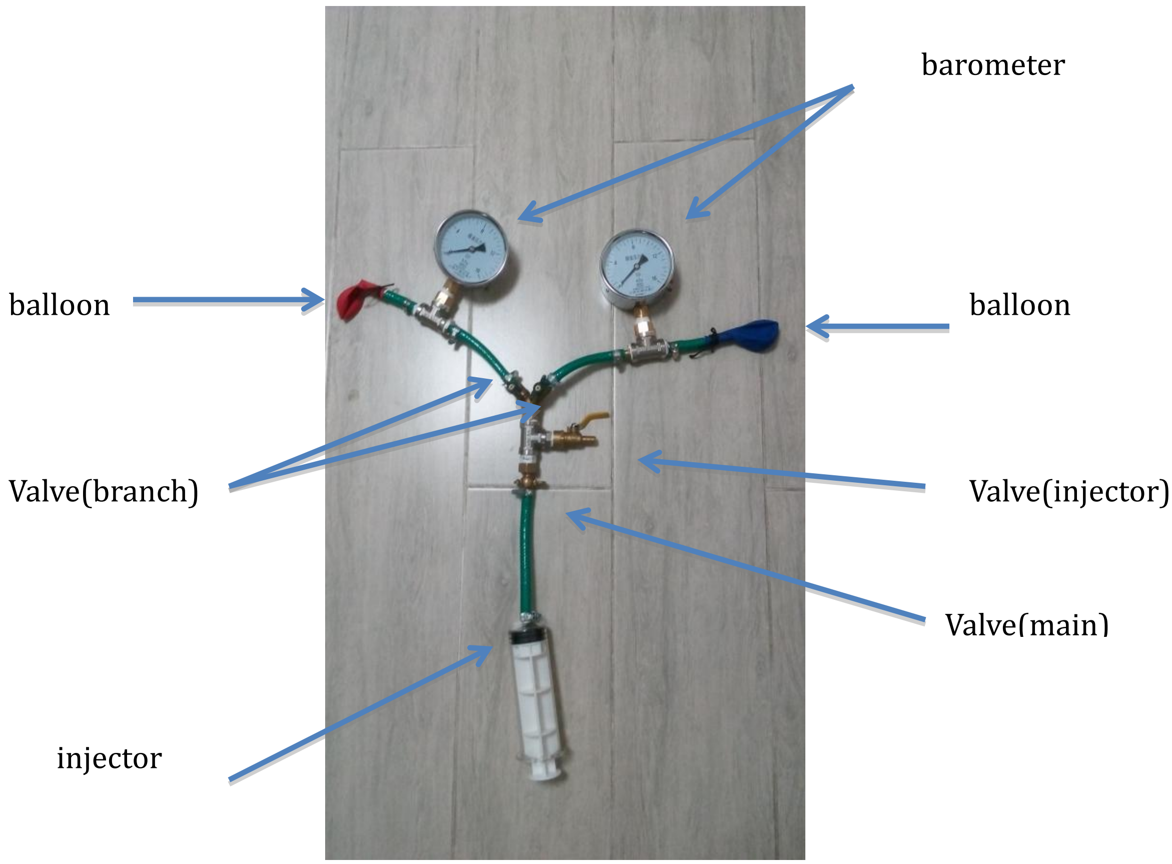
where A and B denote the two balloons respectively. Switching into

volume relation we have $V_0=V_A\left[\dfrac{\left(\dfrac{V_A}{V_B}\right)^{\frac{1}{3}}-1}{\left(\dfrac{V_A}{V_B}\right)^{\frac{7}{3}}-1}\right]^{\frac{1}{2}}$

# 5. Experiment

## 5.1 Experiment setup

The following picture refers to a modified T-connecter, the main equipment used in the experiment:




Pic. 10

Here, the valves on the branch control the flow of air on each branch; the injector valve links the system to the surrounding air and controls the air flow of the injector; the main valve controls the air flow of the entire system. For the first part of the experiment, we use only one branch of the equipment setup and determine the p-r relationship of the balloon. For the second part of the experiment, we will do the two-balloon experiment and observe air flow.

## 5.2 Experimental methods-pressure curve

First of all we will determine the value of $V_0$, or equivalent $r_0$.

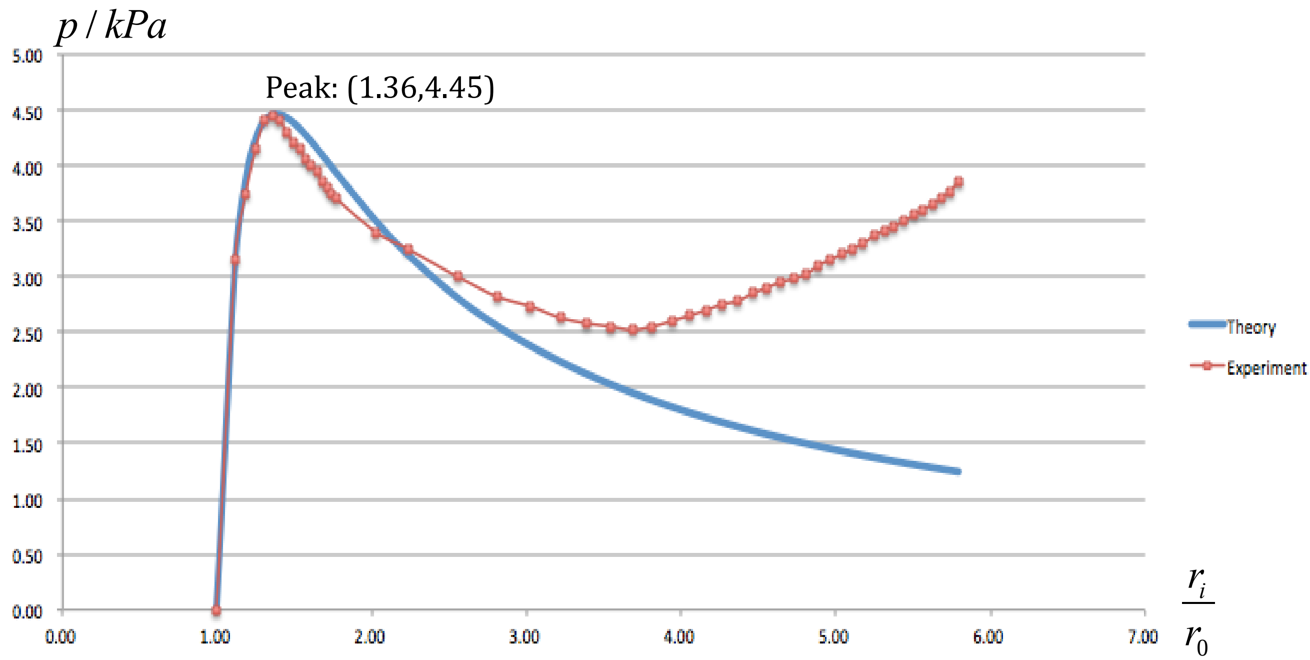
We use the injector to pump air into one of the balloons until the barometer starts to show readings. At that moment the volume inside the balloon can be considered $V_0$.

After determining $V_0$, we pump more air into the balloon, record the volume of the air $V_i$ and barometer reading $p_i$ at the same time. Note that $\frac{V_i}{V_0} = \left(\frac{r_i}{r_0}\right)^3$, so we can thus determine $p - \frac{r_i}{r_0}$ relationship and compare it with theoretical value.

## 5.3 Experiment data of the pressure curve

By experiment, we found out that the initial volume of the balloon is 36mL. And the diagram below represents the comparison between theoretical and experimental values.

We can see from the diagram that under small inflation the theory fits well, but under large inflation the experimental value is significantly larger than the theoretical value.




Pic. 11

## 5.4 Results Analysis

We find out that the pressure arrives at its peak when the radius ratio equals 1.36, which is close to the theoretical value of 1.38. Yet when radius ratio goes beyond 2.5 the experimental value diverts from theoretical curve and forms a lowest point when the ratio is roughly 3.7.

The reason for such a difference is that the balloons used in the experiment are **non-ideal balloons.** A number of factors can affect the pressure curve under large inflation: crystallization, imperfect flexibility of molecular chains, and hysteresis. Under large inflation, the scenario will be very complicated have some degree of randomness. It will be hard to tell the exact value of the pressure curve.

## 5.5 Experiment methods and result-two balloons experiment

In this part we will connect two balloons via the T-connector, record the volume and pressure, and also the direction of air flow. The results are listed below. Here, $p$ denotes the pressure when the system achieves equilibrium.

To avoid the inaccuracy caused by fatigue of balloons, we **change balloons** whenever we want to acquire new sets of data.

| group | $V_1$/mL | $p_1$/kPa | $V_2$/mL | $p_2$/kPa | $p$/kPa | direction |
|---|---|---|---|---|---|---|
| 1 | 400 | 2.70 | 1000 | 2.30 | 2.05 | Small to big |
| 2 | 400 | 2.70 | 1600 | 2.15 | 1.95 | Small to big |
| 3 | 400 | 2.70 | 2000 | 2.20 | 2.00 | Small to big |
| 4 | 400 | 2.70 | 2400 | 2.10 | 2.00 | Small to big |
| 5 | 2000 | 2.20 | 6400 | 3.40 | 2.00 | Big to small |
| 6 | 2500 | 2.00 | 4200 | 2.80 | 1.90 | Big to small |
| 7 | 1800 | 2.30 | 4800 | 3.00 | 2.15 | Big to small |
| 8 | 60 | 3.60 | 130 | 4.00 | 4.30 | Big to small |
| 9 | 200 | 3.35 | 6600 | 3.35 | 3.35 | Equilibrium |
| 10 | 300 | 3.20 | 5400 | 3.20 | 3.20 | Equilibrium |

We can see from the results that the air always flows from the balloon with the bigger pressure to the one with the smaller pressure and at the end of the process, the air pressures inside the two balloons are equal, despite difference in balloon sizes. Normally, when air from big balloon to small one, the final volumes tend to be the same, whereas if not, the difference between volumes will get bigger, but the air pressure will be the same. When the initial pressures inside the balloons are equal, no air flow will occur. The results accord with our previous assumptions and experimental pressure curve.

# 6. Results

In the two-balloon experiment, air always flows from the balloon with the bigger pressure to the one with the smaller pressure. In ideal conditions, as the balloon expands, the pressure first increases, then decreases, reaching a peak when the radius equals 1.38 times the initial radius. But for non-ideal balloons, when the radius is big enough, the pressure will rise again, reaching a lowest point when the radius roughly equals 3.7 times the initial radius, but the process is complicated, and has some degree of randomness.

Depending on initial volumes and radii, air can flow from the bigger balloon to the smaller one, or from the smaller to the bigger, or remain static. All three scenarios are demonstrated in the experiment.

# 7. Error analysis

1. The accuracy of the barometer may affect the accuracy of the results.

2. Minor leakage of air into the environment is inevitable, which will cause some, but not significant, inaccuracy in results.

3. When air is injected into the balloons, its pressure and temperature will change slightly, which may lead to minor inaccuracies.

4. Friction inside the equipment exists, but is small enough to avoid significant errors in result.

# 8. References

1. Wikipedia: Two balloons experiment,

http://en.wikipedia.org/wiki/Two-balloon_experiment

2. F. Weinhaus and W. Barker. On the equilibrium states of interconnected bubbles or balloons. Am. J. Phys. 46, 978-982 (1978)